ONLINE APPENDIX TO
Risk Classification in Insurance Markets with Risk and Preference Heterogeneity

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Abstract

This appendix has two parts. The first provides additional discussion of the results from numerical simulations of our model. The second one shows that the signal monotonicity property proposed in the main paper is equivalent to a statistical property related to the impact signal realizations have on the expected risk assessment of small risk pools. We also discuss the relationship of this property with our equilibrium characterization.

1 Numerical simulations

This section provides additional simulations and results obtained from the price iteration algorithm described in Section 4.2 and illustrated in Section 5.5 of the main text. We first revisit the simulation exercises presented in Section 5.5 and provide additional results. Second, we present results for simulation involving a finer 30-value grid for risk aversion levels. Finally, we present an additional exercise, which explores comparative statics with respect to dispersion in both risk and risk preferences.

1.1 Baseline example - Two levels of risk aversion

First, we revisit the example highlighted in Figure 3 of Section 5.5. This example considers a grid of 500 values for risk levels \( \mu \) in the interval \([3, 7]\) and a binary grid for risk aversion levels \( \rho \in \{8, 9.7\} \). Motivated by Einav et al. (2013), the distribution of types within this grid is a rescaling of the density of a lognormal distribution with:

\[
(\log(\tilde{\mu}), \log(\tilde{\rho})) \sim N \left( \begin{bmatrix} 1.5094 \\ 2.0545 \end{bmatrix}, \begin{bmatrix} 0.2 & -0.12 \\ -0.12 & 0.25 \end{bmatrix} \right).
\]

The distribution of risks, conditional on each level of risk aversion, is plotted in the left panel of Figure 1. The simulated type assignment functions for both risk aversion levels and equilibrium prices are plotted in the right panel of Figure 1. It closely matches the qualitative properties obtained from Proposition 1 and illustrated in Figure 1 of the main text. Extreme coverage levels are purchased by agents with a single level of risk aversion, akin to separating regions, and an intermediary region of coverages combines types with both levels of risk aversion.

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The distinction between the discrete and continuous pooling regions is less obvious as the discreteness of the type and contract sets imposed by the algorithm implies that all traded contracts have a positive mass of types. Figure 2 plots the mass of types purchasing each coverage level. Two particular coverage levels (38% and 80%) have a larger mass of agents than their neighboring levels, which seems like a discrete version of the continuous pooling regions characterized in Proposition 1. Extremely high coverage levels are purchased by a very small mass of agents, e.g., 0.03% agents purchase 100% coverage. This is also in line with Proposition 1, which shows that only high risk aversion agents purchase high coverage levels and that the corresponding type assignment function in this region, \(m_{h}\), has a closed form solution which coincides with its one-dimensional counterpart. This closed form solution has the property that \(m'_{h}(1) = 0\), meaning that the mass of agents choosing contracts in \([1 - \epsilon, 1]\) is of order \(o(\epsilon^2)\).

Figure 1: Simulations with two possible levels of risk aversion. Left panel: risk distribution, conditional on risk aversion. Right panel: type assignment and price per unit from simulated data.

Figure 2: Simulations with two possible levels of risk aversion. Mass of agents pooled in each contract.
1.1.1 Non-monotonic signal example

We now illustrate the results for the disclosure of a non-monotonic binary signal structure, using the same parametrization from Subsection 1.1. Figure 3 displays the impact of signal disclosure on prices (left panel) and interim welfare, as measured by the equivalent variation to signal disclosure (right panel). The left panel plots equilibrium prices per unit of coverage prior to the signal realization and contingent on both possible realizations. We can see that signal disclosure has an almost zero effect on equilibrium prices. As a consequence, the equivalent variation of signal disclosure is approximately zero for all consumers (we have used the same scale used in figure 3 in the main text for the sake of comparison).

These results can be directly compared to Figure 3 in the main text, which shows a non-trivial price and positive welfare effect from the disclosure of a monotonic signal. The comparison of these two simulations shows that the monotonic signal leads to more welfare gains for almost all types (except for the very top types, which have negligible welfare changes). This is in line with the more general pattern presented in Figure 4, where the share of types with interim welfare improvements is higher with a monotonic signal structure.

1.2 Multiple levels of risk aversion

We now consider a model simulation with a $30 \times 30$ grid of the type space for $(\mu, \rho)$, with the range of types and distribution otherwise identical from the simulation highlighted in 1.1. The left panel of Figure 4 plots the distribution of risk levels $\mu$, conditional on risk aversion $\rho$, for a few levels of risk aversion in the grid. The left panel of Figure 5 plots the type assignments for the extreme levels of risk aversion in the grid, together with the price per unit of each contract. Both curves represent, for each coverage $x$, the upper and lower bounds for the set of risk levels pooled at $x$. Differently from the simulation in the last section, contracts now are purchased by agents with multiple levels of risk aversion. The right panel of figure 5 contains a heatmap representing the coverage level chosen by each type in the grid. The set of types pooled together at each coverage level are represented by regions of same color.
1.3 Risk dispersion

Another interesting question is the relative relevance of dispersion in both risk and risk aversion. To study this issue, we have performed simulations over a grid of parameters, fixing the mid level of risk and risk aversion ($\mu_0 = 5$ and $\rho_0 = 8$, respectively), and varying the dispersion around this mid points (represented by $\delta$ and $\Delta \mu$). Similarly to the exercise discussed in the paper, we have analyzed the cases of monotonic and non-monotonic signal structures, as well as considering both a binary as well as a finer 30-element grid for risk aversion levels.

Figure 6 presents the interim welfare improvement results for a finite grid with 500 values for risk levels and two levels for risk aversion, for each choice of $\delta$ and $\Delta \mu$. The left-hand side uses a monotonic signal structure. We can see that, as expected, the share of improving agents approaches one as preference dispersion becomes small. On the other hand, the effect of risk dispersion on signal analysis seems less clear.

Figure 7 illustrates similar simulation outcomes, but instead looking at a $30 \times 30$ grid for types ($\rho, \mu$). Once again, the left panel uses a monotonic signal structure, while the right panel utilizes a non-monotonic quadratic signal structure. The general pattern of improvements is similar to the ones for the binary risk aversion case illustrated in Figure 6 but less noisy. As discussed in the main text, a possible interpretation of this discrepancy is that the model with two levels of risk aversion is more susceptible to the issue of equilibrium multiplicity.
signal realizations have on the expected risk assessment of a small risk pool. Its connection with equilibrium 
agents when varying both the dispersion in risk (x axis) and risk aversion (y axis). Left Panel: monotonic signal 
structure. Right panel: non-monotonic quadratic signal structure.

Nonetheless, the shared qualitative features of both sets of simulations is reassuring.

Figure 7: Numerical simulations using 30 × 30 grid for risk and risk aversion. Share of interim improving 
agents when varying both the dispersion in risk (x axis) and risk aversion (y axis). Left Panel: monotonic signal 
structure. Right panel: non-monotonic quadratic signal structure.

2 The statistical content of monotonicity

In this section we show that monotonicity is equivalent to a simple statistical property related to the impact 
signal realizations have on the expected risk assessment of a small risk pool. Its connection with equilibrium 
analysis is discussed below.

Consider a set or pool of types with different levels of risk aversion and risk heterogeneity $\varepsilon > 0$, defined by:

$$ T (\overline{\mu}, \varepsilon) = \{ (\overline{\mu}, \rho_i) : (\overline{\mu} + \varepsilon, \rho_i) \} , $$

and denote the expected risk level in this set as its cost $C$. For a signal structure $\pi (\cdot | \cdot)$, the impact of signal 
realization $s \in S$ on the cost of pool $T$ is given by

$$ \Delta C^s (\overline{\mu}, \varepsilon) = E \left[ \overline{\mu} | (\overline{\mu}, \rho) \in T (\overline{\mu}, \varepsilon), \tilde{s} = s \right] - E \left[ \overline{\mu} | (\overline{\mu}, \rho) \in T (\overline{\mu}, \varepsilon) \right] . $$
Our equilibrium characterization shows that the price of a given coverage \( x \) is indirectly affected by the type distribution within pools with coverage levels above \( x \). This top-down property of equilibrium prices implies that any changes in the cost of a pool \( T \) will indirectly affect the prices of lower coverage contracts consumed by consumers with lower risk level \( \mu < \bar{\mu} \). We now focus on the indirect expected effect that signal disclosure has on consumers with risk level \( \mu \) through its impact on the cost of pool \( T \):

\[
\Delta C (\bar{\mu}, \mu; \varepsilon) \equiv \sum_{s \in S} \pi (s \mid \mu) \Delta C^* (\bar{\mu}, \varepsilon).
\]

The result below shows that monotonicity is equivalent to negativity of the indirect cost effect described here.

**Proposition 1.** A signal structure \( \pi \) is monotonic if, and only if, for any continuous type distribution \((\phi_l, \phi_h)\) with support \([\mu_L, \mu_H]\), almost all \( \mu, \bar{\mu} \in (\mu_L, \mu_H) \) and \( \varepsilon > 0 \) sufficiently small,

\[
\Delta C (\bar{\mu}, \mu, \varepsilon) < 0.
\]

**Proof.** For brevity, define \( \pi_0 (\cdot) \equiv \frac{1}{\#S} \). We then have that, for \( k \in S \cap \{0\} \),

\[
\Delta C^* (\bar{\mu}, \varepsilon) \equiv \varepsilon \frac{\pi (s \mid \bar{\mu} + \varepsilon) \phi_l (\bar{\mu} + \varepsilon)}{\pi (s \mid \bar{\mu} + \varepsilon) \phi_l (\bar{\mu} + \varepsilon) + \pi (s \mid \bar{\mu}) \phi_h (\bar{\mu})}.
\]

\[
\Delta C (\bar{\mu}, \mu, \varepsilon) = \varepsilon \sum_{s \in S} \pi (s \mid \mu) \frac{\pi (s \mid \bar{\mu} + \varepsilon) \phi_l (\bar{\mu} + \varepsilon)}{\pi (s \mid \bar{\mu} + \varepsilon) \phi_l (\bar{\mu} + \varepsilon) + \pi (s \mid \bar{\mu}) \phi_h (\bar{\mu})} - \frac{\phi_l (\bar{\mu} + \varepsilon)}{\phi_l (\bar{\mu} + \varepsilon) + \phi_h (\bar{\mu})}
\]

\[
= \varepsilon \sum_{s \in S} \pi (s \mid \mu) \int_0^\varepsilon \frac{\phi_l (\bar{\mu} + z)}{\phi_l (\bar{\mu} + z) + \frac{\pi (s \mid \bar{\mu})}{\pi (s \mid \bar{\mu} + z)} \phi_h (\bar{\mu})} dz.
\]

These imply that

\[
\lim_{\varepsilon \to 0} \frac{\Delta C (\bar{\mu}, \mu, \varepsilon)}{\varepsilon^2} = \sum_{s \in S} \pi (s \mid \mu) \frac{\partial}{\partial z} \left[ \frac{\phi_l (\bar{\mu} + z)}{\phi_l (\bar{\mu} + z) + \frac{\pi (s \mid \bar{\mu})}{\pi (s \mid \bar{\mu} + z)} \phi_h (\bar{\mu})} \right] \bigg|_{z=\varepsilon=0}
\]

\[
= \sum_{s \in S} \pi (s \mid \mu) \frac{\hat{\pi} (s \mid \bar{\mu}) \phi_l (\bar{\mu}) \phi_h (\bar{\mu})}{\pi (s \mid \bar{\mu}) [\phi_l (\bar{\mu}) + \phi_h (\bar{\mu})]^2},
\]

which is strictly negative, for any continuous distribution \((\phi_l, \phi_h)\) with support \([\mu_L, \mu_H]\) and almost all \(\mu_L < \mu < \mu_H\) if, and only if, \(\sum_{s \in S} \pi (s \mid \mu) \frac{\hat{\pi} (s \mid \bar{\mu})}{\pi (s \mid \bar{\mu})} < 0\) for almost all \(\mu_L < \mu < \mu_H\), i.e., monotonicity holds.

While Proposition 1 does not use equilibrium objects, these two are connected since, in equilibrium, prices are determined by the average riskiness of risk pools and the top-down property of prices is represented by the indirect cost assessment introduced here.